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EFFECTS OF SOLAR RADIATION PRESSURE UPON SATELLITE ATTITUDE CONTROL

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ABSTRACT

This paper discusses the effects of torques due to solar radiation pressure acting on satellite vehicles. The functional forms of the torques are developed for inclusion in the general equations of motion of the vehicle. These combined equations are highly nonlinear; however, with certain restrictions, approximate linear solutions are derived for the motion of the vehicle due to the action of the solar radiation torques.

The torque equations are highly dependent upon the vehicle configuration, the reflective properties of the surfaces exposed to the sun's rays, and the vehicle orientation with respect to a sun reference. The expressions are developed for two cases of practical interest; spin-stabilized vehicles, and vehicles with solar arrays and an active attitude control system which orients the vehicle in an earth-sun reference. Similar expressions for other configurations can be developed through use of the methods discussed herein.

The results of this analysis indicate the solar radiation pressure may cause torques that are either periodic or constant with respect to inertial space, depending on the orbital inclination to the ecliptic plane, the orientation requirements and the vehicle configuration. The approximate solutions, developed here can be used to determine the vehicle angular momentum storage and/or removal requirements resulting from solar radiation torques.

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INTRODUCTION

Many applications of satellites requiring attitude control have been discussed in scientific literature. In general, disturbance torques will act to produce deviations of the vehicle from the desired attitude. A number of the various sources of these attitude perturbation torques are summarized in Reference 1. This paper is concerned with a more detailed discussion of the effects of one particular disturbance torque source, solar radiation pressure.

The model chosen to represent the solar pressure on a vehicle surface is one in which the force on the surface due to emission is neglected, the angle of reflection is assumed to equal the angle of incidence, and the coefficient of reflectivity for the surface is assumed independent of the spectrum or angle of incidence of the incident radiation. Torques will then act about the vehicle center of mass if the resultant center of pressure of the total solar radiation pressure forces is not coincident with the vehicle center of mass. The torque acting on the satellite from solar radiation pressure is therefore a function of the vehicle orientation with respect to the solar radiation field and the relative location and characteristics of the exposed satellite surfaces.

SOLAR RADIATION TORQUE ON AN ARBITRARY VEHICLE

A typical satellite vehicle may be divided into several distinct surfaces, each with a characteristic area and surface reflective properties. The force on an arbitrary surface is due to the reaction from absorption and reflection of the radiation on the particular surface. Both magnitude and direction of this force are affected by the surface properties. The torque about the vehicle center of mass will be due to the forces on the respective surfaces acting through a center of pressure determined by surface characteristics and geometry.

The unit vector $\hat{v}$ is defined as the vector directed from the vehicle center of mass to the sun. For a vehicle orbiting the earth, this vector may be assumed coincident with the vector directed from the center of the earth to the sun. The force due to the solar radiation incident upon a surface A_i of a vehicle may be expressed by:

$$\overline{F}_i = - V\left[\int_{A_i} (1+\nu_i) \left| \hat{v} \cdot \hat{n} \right| (\hat{v} \cdot \hat{n})\hat{n} dA_i + \int_{A_i} (1-\nu_i) \left|\hat{v}\cdot\hat{n}\right| \hat{n}x\hat{v}x\hat{n} \, dA_i \right] \quad (1)$$

where ν is the coefficient of reflectivity over the surface, $\hat{n}$ is a unit vector directed along the outward normal to dA_i, and V is the solar radiation pressure constant[1] for normal incidence ($|\hat{v}\cdot\hat{n}| = 1$) and complete absorptivity ($\nu = 0$). The total solar radiation torque about the vehicle center of mass is then:

$$\overline{T} = \sum_{A_i} \overline{\ell}_i x \overline{F}_i \quad (2)$$

where $\overline{\ell}_i$ is the vector directed from the vehicle center of mass to the center of pressure of the i^{th} surface.

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[1] The solar radiation energy constant outside the earth's atmosphere is $1.94 \frac{gm\ cal}{cm^2 min}$ [Reference 2] , corresponding to a pressure of $9.4 \times 10^{-8} lb/ft^2$

COORDINATE SYSTEMS AND NOTATION

Definition of several reference coordinate sets is convenient in order to express the orientation of the vehicle relative to inertial space and to derive the solar radiation torques using Eq. (2). These coordinate sets are shown in Figure 1, and are defined as follows:

1 <u>Ecliptic Reference</u> $(x_i y_i z_i)$ - An inertial coordinate set with origin at the center of the earth; the $x_i z_i$ plane the ecliptic plane; and the z_i axis along the line of equinoxes, directed toward the autumnal equinox.

2. <u>Orbital Reference</u> $(x_o y_o z_o)$ - An inertial coordinate set with origin at the center of the earth; the $x_o z_o$ plane the orbit plane; and the z_o axis along the line of nodes, directed toward the ascending node. The angles ξ and β are respectively the inclination of the orbit plane to the ecliptic and the angle in the $x_i z_i$ plane between the z_i and the z_o axes.

3. <u>Sun Reference</u> $(x_o' y_o' z_o')$ - A slowly rotating coordinate set with origin at the center of the earth; the $x_o' z_o'$ plane the orbit plane; and the $y_o' z_o'$ plane contains the sun vector, $\hat{v}$. The angles S' and η are respectively the angles between the y_o' axis and $\hat{v}$, and the angle between the z_o and z_o' axes.

4 <u>Vehicle Reference</u> $(x_r y_r z_r)$ - A rotating coordinate set with origin at the vehicle center of mass (c.m.); the $x_r z_r$ plane the orbit plane; and the z_r axis directed along the vector from the center of the earth to the vehicle c.m., $\bar{r}_o$. The angle $\alpha(t)$ is the angle between $\bar{r}_o$ and the z_o axis

The unit vector, $\hat{v}$, lies in the ecliptic plane; its position relative to the vernal equinox ($-z_i$ axis) is given by the angle S When resolved to the vehicle reference set, $\hat{v}$ is given by:

$$\begin{bmatrix} v_{xr} \\ v_{yr} \\ v_{zr} \end{bmatrix} = \begin{bmatrix} -\sin(\alpha - \eta)\sin S' \\ \cos S' \\ \cos(\alpha - \eta)\sin S' \end{bmatrix} \tag{3}$$

where S' and η are defined from the geometry by

$$\cos S' = \sin\xi \sin(S - \beta) \tag{4}$$

$$\tan \eta = \frac{-\cos\xi \sin(S - \beta)}{-\cos(S - \beta)} \tag{5}$$

(The minus sign denotes the choice of the proper quadrant for η).

The body fixed coordinate set XYZ is defined with origin at the vehicle center of mass The XYZ coordinate set may be chosen to be the principal axes of the vehicle if convenient. For an attitude controlled satellite, the desired orientation of the vehicle relative to a convenient reference is known as a function of time. Since many applications of earth satellites require alignment of a body fixed reference with the radius vector, $\bar{r}_o$, a convenient choice for a reference is the $x_r y_r z_r$ coordinate set. The orientation of the body set relative to the $x_r y_r z_r$ set may be uniquely defined by three angles, ψ, θ, and ϕ. These angles are represented by ordered rotations about the z_r, the y_r', and the x_r'' axes, where the primes refer to the new orientation of the coordinate axes after rotations. The matrix $[A]$ which resolves vectors in the $x_r y_r z_r$ set to the body set is given by:

$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{6}$$

where

$$\begin{aligned}
a_{11} &= \cos\theta \cos\psi \\
a_{12} &= \cos\theta \sin\psi \\
a_{13} &= -\sin\theta \\
a_{21} &= -\sin\psi \cos\phi + \sin\phi \sin\theta \cos\psi \\
a_{22} &= \cos\phi \cos\psi + \sin\phi \sin\theta \sin\psi \\
a_{23} &= \sin\phi \cos\theta \\
a_{31} &= \sin\psi \sin\phi + \sin\theta \cos\psi \cos\phi \\
a_{32} &= -\cos\psi \sin\phi + \cos\phi \sin\theta \sin\psi \\
a_{33} &= \cos\theta \cos\phi
\end{aligned} \tag{7}$$

The angles ϕ, θ, and ψ can be written as functions of time or orbital position α to define the prescribed vehicle orientation relative to $x_r y_r z_r$. Then, given the desired orientation angles ϕ, θ, and ψ, the components of $\hat{v}$ can be expressed in body coordinates, and Eq. (2) will give the solar radiation torques for a particular vehicle configuration.

Two particular cases of earth satellites will be considered here: 1) a non-spinning earth-sun oriented satellite that is required to align one axis with the local vertical and point a body fixed solar cell array at the sun, 2) a spin-stabilized vehicle requiring the spin axis to be parallel to the normal to the orbit. Other cases may be investigated using the methods developed here; however, the two examples treated are typical of many earth satellites (communication, surveillance, weather, navigation), hence are felt to be significant.

EARTH-SUN ORIENTED SATELLITE

The vehicle described here is required to stabilize with respect to the sun and the earth. A typical configuration is shown in Figure 2. A body fixed antenna (Z axis) is to be aligned with the z_r axis and the solar paddles (cells on one side only), which are free to rotate ±90 degrees about the X axis, are to be normal to the sun's rays. The control system is assumed to include a means of storing angular momentum with a maximum storage capability, and a means of angular momentum removal. Attitude error signals can be given by various sensing devices (horizon scanners, sun seekers, etc.).

A general approach is developed to determine the momentum storage and removal requirements due to disturbance torques acting on vehicles with this capability. This approach involves resolving the disturbance torques to an inertial reference where the components can be integrated directly to obtain the total angular momentum of the vehicle as a function of time.

Newtons Second Law of Motion written for a rotating coordinate set is:

$$\overline{T} = \dot{\overline{H}} + \overline{\omega} \times \overline{H} \tag{8}$$

where $\overline{T}$, $\overline{H}$, and $\overline{\omega}$ are the disturbance torques, the total vehicle moment of momentum, and the angular velocity, respectively, expressed in vehicle body coordinates. The moment of momentum (angular momentum) is given by:

$$\overline{H} = \overline{\overline{I}}\,\overline{\omega} + \overline{H}_s \tag{9}$$

where $\overline{H}_s$ is the angular momentum of the momentum storage device, and $\overline{\overline{I}}$ is the inertia tensor of the vehicle referred to the body coordinate set. Expressing the disturbance torques, the angular rate, $\overline{\omega}$, and the inertia tensor $\overline{\overline{I}}$ in the body coordinate set gives a set of three linear differential equations with time varying coefficients to solve for $\overline{H}_s$. In general, however, direct solution may be quite difficult.

If the torques can be resolved to an inertial coordinate set, Eq. (8) reduces to:

$$\overline{T}_i = \frac{d\overline{H}_i}{dt} \tag{10}$$

where the subscript i refers to the vectors resolved in an inertial coordinate system. Let the matrix $[B]$ represent a general transformation which resolves the vector from the body coordinate set to any inertial reference set. Then Eq. (10) may be written:

$$\frac{d\overline{H}_i}{dt} = [T_i] = [B]^{-1}[T] \tag{11}$$

and, $\overline{H}_i$ is given by:

$$[H_i] = \int_{t_o}^{t} [B]^{-1}[T] \, d\tau + [H_{oi}] \tag{12}$$

where $\overline{H}_{oi}$ is a constant vector and represents the total angular momentum of the vehicle at $t = t_o$.

To obtain the angular momentum of the vehicle, $\overline{H}$, in body coordinates, the vector $\overline{H}_i$ must be resolved back to body coordinates:

$$[H] = [B] \left\{ [H_{oi}] + \int_{t_o}^{t} [B^{-1}(\tau)] [T\ (\tau)]\ d\tau \right\} \tag{13}$$

Since the orbital position α is a function of time, Eq. (13) can be written:

$$[H(\alpha)] = [B(\alpha)] \left\{ [H_{oi}] + \int_{\alpha_o}^{\alpha} [B^{-1}(\alpha)] [T\ (\alpha)] \frac{d\alpha}{\omega_o} \right\} \tag{14}$$

where $\omega_o = \frac{d\alpha(t)}{dt}$, and is constant for a circular orbit.

In the absence of disturbance torques, the angular momentum of the momentum storage device is given by:

$$\bar{H}_{so} = \bar{\bar{B}}\ \bar{H}_{oi} - \bar{\bar{I}}\ \bar{\omega} \tag{15}$$

The components of $\bar{H}_{so}$ along the body axes can be only constants or periodic functions of time.

The oreintation requirements for the vehicle shown in Figure 2 are described by:

a) Antenna alignment: $\phi = \theta = 0$

b) Solar array alignment: $(\hat{v} \cdot \hat{e}_X) = 0$ (16)

$$(\hat{v} \cdot \hat{e}_{zp}) = 0 \tag{17}$$

$$(\hat{v} \cdot \hat{e}_{yp}) = 1 \tag{18}$$

The paddle angle rotation, ϕ_p, relative to the vehicle body axes is shown in Figure 2 along with the $x_p y_p z_p$ coordinate set. Substitution of Eqs (3), (6), and (7) into (16), (17), and (18) gives the desired yaw and paddle motion:

$$\psi_i = \tan^{-1}\left[\sin(\alpha-\eta)\tan S'\right] \qquad 0 \leq \psi \leq 2\pi \tag{19}$$

$$\phi_p = \sin^{-1}\left[\cos(\alpha-\eta)\sin S'\right] \qquad -\frac{\pi}{2} \leq \phi_p \leq \frac{\pi}{2} \tag{20}$$

The solar radiation torques acting on this vehicle together with the components of $\Diamond$ along the body axes are developed in Appendix 1. The case of ideal orientation will be chosen to demonstrate the approach of Eq. (8) through (14). Therefore, the solar radiation torques are given by Eqs. (A-11) and (A-13).

Since the orbital rate, $\frac{d\alpha}{dt} = \omega_o(t)$, is usually much larger than the rate of earth's rotation about the sun, $\frac{dS}{dt} = \omega_k$, the angles S, S', and η may be assumed constant over several orbits. Hence the sun reference coordinate set can be considered an "inertial" coordinate set over several orbits. The matrix $\left[B\right]$ which transforms vectors from the sun reference set to the body set for ideal orientation is:

$$\left[B\right] = \begin{bmatrix} \cos(\alpha-\eta)\cos\psi_i & \sin\psi_i & -\sin(\alpha-\eta)\cos\psi_i \\ -\sin\psi_i\cos(\alpha-\eta) & \cos\psi_i & \sin(\alpha-\eta)\sin\psi_i \\ \sin(\alpha-\eta) & 0 & \cos(\alpha-\eta) \end{bmatrix} \tag{21}$$

The assumption of ideal orientation implies that all changes of angular momentum must be stored in the momentum storage device in order to maintain ideal orientation. This angular momentum change in inertial space due to the solar torques is given by substitution of Eqs. (21), (A-11), and (A-13) in the expression:

$$\left[H_{si}\right] = \int_{\alpha_o}^{\alpha} \left[B\right]^{-1} \left[T\right] \frac{d\alpha}{\omega_o} \tag{22}$$

where $\bar{H}_s = \bar{H}_{so} + \bar{\bar{B}}\,\bar{H}_{si}$ gives the momentum of the momentum storage device in body coordinates.

For a circular orbit this results in terms periodic with frequencies ω_o and $2\omega_o$, and terms which increase monotonically with time. The maximum amplitude of the periodic terms gives the minimum momentum storage capability requirements. Integration of Eq. (22) over one orbit gives the average build-up of angular momentum per orbit as:

$$\Delta\bar{H}/\text{orbit} = \Delta H_{xo'}\,\hat{e}_{xo'} = -\left\{\frac{4L_Y \cos S'}{\omega_o}\left\{\gamma_{pn}\,\frac{K(|\sin S'|) - E(|\sin S'|)}{\sin S'}\right.\right.$$
$$+ \gamma_{en}\,\frac{|\sin S'|}{2\sin S'}\left[-1 + \frac{1}{2|\sin S'|}\left(1+\sin^2 S'\right)\ell n\,\frac{(1+|\sin S'|)}{(1-|\sin S'|)}\right]\Bigg\}$$
$$\left. + \frac{\pi \sin S' \cos S'}{\omega_o}\left[(L_Y + R_o)\gamma_{ct} - \frac{h}{2}\,\gamma_{et}\right]\right\}\hat{e}_{xo'} \qquad (23)$$

where

$A_e V(1+\nu_e) = \gamma_{en}$, $2A_p V(1+\nu_p) = \gamma_{pn}$, $A_e V(1-\nu_e) = \gamma_{et}$, and $A_c V(1-\nu_c) = \gamma_{ct}$.

The sun reference set cannot be assumed an inertial reference set for times longer than several orbits. Summation of Eq. (23) in an inertial reference using the angle η over one year shows several components of the equation are periodic with frequency of $2\omega_k$. However if the momentum storage device is required also to store this low frequency periodic momentum, the minimum storage capability requirements are increased by approximately the factor $\omega_o/2\omega_k$. This may or may not be desirable with power and weight considerations, depending on the desired satellite lifetime. If it is not desirable, the momentum removal requirements per year will be the sum of the magnitudes of the $\Delta\bar{H}$ over a year.

For this case, several observations can then be made about the momentum removal requirements by investigation of Eq. (23):

1) When the sun lies in the orbit plane, $(S' = \frac{\pi}{2})$, no angular momentum removal is necessary.

2) When the sun is always normal to the orbit plane $(S' = 0)$, no angular momentum removal is necessary. However, this can occur only for an orbit plane normal to the ecliptic plane and regressing at the rate of the earth's rotation about the sun, ω_k.

3) The momentum added per orbit is dependent only upon the c.m. -c.p. vector component along the Y axis.

A typical set of vehicle parameters is chosen to demonstrate the requirements for the removal of angular momentum for an equitorial orbit ($\xi = 23.45^\circ$). The parameters are:

$L_Y = 0.2$ ft	$A_e = 12$ ft^2	$A_p = 100$ ft^2
$A_c = 20$ ft^2	$\nu_p = 0.40$	$\nu_c = \nu_e = 0.95$
$R = 1.75$ ft	$h = 5.0$ ft	$\frac{2\pi}{\omega_o} = 24$ hrs.

The magnitude of $\overline{\Delta H}$ is plotted in Figure 3 for one quarter of a year. The total momentum removal requirement per year is then four times the area under the curve of Figure 3, or 25.2 ft-lb-sec per year.

The example discussed here considered one particular scheme of earth-sun orientation, namely paddle motion constrained to $\pm\frac{\pi}{2}$ and yaw motion unconstrained. The extension of these results to other vehicle configurations and orientation requirements changes only the form of $\overline{T}$, which may be derived as in Appendix 1. The extension to elliptic orbits requires the inclusion of the orbital rate as a function of time in Eq. (22).

SPIN STABILIZED SATELLITE

Spin stabilization is an effective means of attitude control provided that an orientation fixed in inertial space is desired. The effects of disturbance torques will introduce a wobble motion of the spin stabilized vehicle. The vehicle chosen as an example here is a cylindrical vehicle spinning about its polar axis (see Figure 4), and required to maintain the spin axis normal to the orbit plane (Z parallel with y_o'). The allowable deviation from the nominal orientation and the magnitude of the wobble will determine what type of control is required. For example, if the wobble motion is bounded within the allowable deviation cone, no control system will be necessary; if the total angular momentum vector ($\overline{H}$) motion is bounded and within the allowable cone, but the wobble is not, a simple passive damper which aligns the spin axis with the $\overline{H}$ vector will suffice; if neither condition will satisfy, an active control system will be required to fulfill the orientation requirements by controlling the $\overline{H}$ vector.

The wobble motion due to the action of the solar radiation torques is developed. The terms in the disturbance torques dependent on the small angular deviations of the spin axis from the nominal and the deviations from the nominal spin speed relative to the x_r y_r z_r set, ω_s, are neglected.

The expression for the solar radiation torques on the vehicle is obtained from Eq. (A-11) of Appendix 1 by neglecting the solar paddle terms ($A_p = 0$). However, the expressions for $\hat{v}$ will differ, for the vehicle Z axis is to be aligned with the y_o' axis rather than the z_r axis as in the first example. This change is easily made by a simple rotation of $-\frac{\pi}{2}$ about the x_r axis before resolving $\hat{v}$ to the body coordinate set.

This results in:

$$\begin{bmatrix} v_X \\ v_Y \\ v_Z \end{bmatrix} \cong \begin{bmatrix} -\sin S' \sin(\omega_r t - \eta) \\ -\sin S' \cos(\omega_r t - \eta) \\ 0 \end{bmatrix} \tag{24}$$

where $\omega_r t = (\omega_o + \omega_s) t$. Eq. (24) is substituted in Eq. (A-11) to give the solar radiation torque for this case.

Equation (12) will be used to determine $\overline{H}_i (t)$; however the matrix $[B]$ for this case will be given by:

$$[B] \cong \begin{bmatrix} \cos(\omega_r t - \eta) & 0 & -\sin(\omega_r t - \eta) \\ -\sin(\omega_r t - \eta) & 0 & -\cos(\omega_r t - \eta) \\ 0 & 1 & 0 \end{bmatrix} \tag{25}$$

Substitution of $[B]^{-1}$ in Eq. (12) along with the torque expressions gives the expression for $\overline{H}_i (t)$. Eq. (B-8) of Appendix 2 gives the resulting expression for $\overline{H}_i (t)$.

For a small orbital inclination $(\xi \to 0)$ Eq (B- 8) reduces to:

$$\begin{bmatrix} H_{xo}(t) \\ H_{yo}(t) \\ H_{zo}(t) \end{bmatrix} \cong \begin{bmatrix} \dfrac{L_Z V (1+\nu_c) 2Rh}{\omega_k} \sin \omega_k t \\ I_Z \omega_r \\ \dfrac{L_Z V (1+\nu_c) 2Rh}{\omega_k} (\cos \omega_k t - 1) \end{bmatrix} \tag{26}$$

For a vehicle with a passive damper, Eq. (26) describes the motion of the spin axis of the vehicle in inertial space. This motion of the vehicle spin axis (Z) will trace a cone passing through the y_o' axis with frequency ω_k and half cone angle of

$$\frac{L_Z V (1+\nu_c)\, 2R\, h}{\omega_k\, I_Z\, \omega_r}$$

However, with no passive damping in the vehicle, the spin axis will not be coincident with the total angular momentum vector, $\overline{H}$. The equation of motion of the vehicle [Eq. (13)], assuming that deviations of the spin axis from the nominal position are small, gives:

$$\begin{bmatrix} I_X (\dot{\phi} - \omega_r \theta) \\ I_Y (\dot{\theta} + \omega_r \phi) \\ I_Z \omega_r \end{bmatrix} \cong \begin{bmatrix} H_{xo}(t) \cos \omega_r t - H_{zo}(t) \sin \omega_r t - \theta I_Z \omega_r \\ -H_{xo}(t) \sin \omega_r t - H_{zo}(t) \cos \omega_r t + \phi I_Z \omega_r \\ I_Z \omega_r \end{bmatrix} \tag{27}$$

where ϕ and θ are deviation angles as seen by a body fixed observer. Solving (27) for $\phi(t)$ and $\theta(t)$ and resolving the vector $\hat{e}_Z$ into the orbital reference set gives the motion of the spin axis in inertial space as:

$$\begin{bmatrix} \hat{e}_Z \cdot \hat{e}_{xo} \\ \\ \hat{e}_Z \cdot \hat{e}_{zo} \end{bmatrix} \cong \frac{L_Z V (1+\nu_c)\, 2R\, h}{I_Z} \begin{bmatrix} \dfrac{\sin \omega_k t}{\omega_k \omega_r} - \dfrac{\sin (I_Z/I) \omega_r t}{\omega_r^2 I_Z/I} \\ \\ \dfrac{(\cos \omega_k t - 1)}{\omega_r \omega_k} - \dfrac{\cos (I_Z/I) \omega_r t}{\omega_r^2 I_Z/I} \end{bmatrix} \tag{28}$$

$$\hat{e}_Z \cdot \hat{e}_{yo} \cong 1$$

Hence, compared with Eq. (26), Eq. (28) shows the free wobble motion to be a cone about the $\overline{H}$ vector with frequency $(I_Z/I)\,\omega_r$ and half cone angle

$$\frac{IL_Z\, V\,(1+\nu_c)\, 2R\ h}{I_Z^2\ \omega_r^2}$$

For this particular case, the effect of solar radiation torques on the spin stabilized vehicle results in a bounded wobble motion of known magnitude. As the spin frequency is increased, the deviation from the nominal orientation approaches zero as a limit, hence the required spin frequency for a given maximum deviation can be calculated. However, as seen in Eq. (B-8) of Appendix 2, the $\overline{H}$ vector motion will become unbounded for $\xi \neq 0$, and the rate of buildup of the deviation from the y_o axis will increase as the inclination increases. However, this rate can be minimized by minimizing the parameter $(\nu_c - \nu_e)$, or by increasing the spin frequency ω_r. From Eq. (28) it is seen that increasing the spin frequency will also decrease the bounded wobble motion.

The extension to other configurations and orientation requirements follows directly from this example and the appendices.

APPENDIX 1

Solar Radiation Torque for an Earth-Sun Oriented Satellite

Equation (2) of the text may be used to derive the expression for the solar radiation torque on the vehicle shown in Figure 2. The following assumptions are made:

1) The time spent by the vehicle in the shadow of the earth is neglected in this development.
2) The paddle control is perfect such that the solar radiation vector is in the plane normal to the face of the paddle containing the paddle control axis. (Yaw attitude control may be imperfect.)
3) The solar paddles are thin such that the edge effects may be neglected, and the paddle c.p. lies at the paddle centroid
4) The paddles cast no shadows on the body and vice versa.
5) The vehicle c.m. does not lie at the centroid; the paddles are symmetrical to the vehicle centroid.

Define the vector $\bar{L}$ fixed in the vehicle as the vector from the vehicle c.m. to the centroid. Then from Assumption (2) the terms $(\hat{v} \cdot \hat{n}) \hat{n}$ and $\hat{n} \times \hat{v} \times \hat{n}$ become for the paddles:

$$(\hat{v} \cdot \hat{n}) \hat{n} = v_Y \hat{e}_Y + v_Z \hat{e}_Z \qquad \text{(A-1)}$$

$$\hat{n} \times \hat{v} \times \hat{n} = v_X \hat{e}_X \qquad \text{(A-2)}$$

Since the c.p. of the two paddle set lies at the centroid, Eq. (the paddle contribution:

$$\overline{T}_p = 2\overline{L} \times V \left[(v_Y \hat{e}_Y + v_Z \hat{e}_Z)\ (1 + \nu_p)\, A_p + v_X \hat{e}_X (1 - \nu_p)\, A_p \right] \sqrt{1 - v_X^2} \tag{A-3}$$

The vehicle body may be broken into two surfaces - the end faces and the cylindrical portion.

The c.p. locations relative to the vehicle center of mass for each surface can be calculated by:

$$\overline{c.p.}_i \times \int_{A_i} \overline{F}\, dA_i = \int_{A_i} (\overline{S} \times \overline{F})\, dA_i \tag{A-4}$$

where $\overline{S}$ is a vector from the vehicle centroid to a point on the i^{th} surface. For the cylindrical body considered here, let both ends be identical and ν_e and ν_c be constant, where the subscripts e and c refer to the end and the curved surfaces respectively. Then, applying Eq. (A-4) the c.p. locations are:

$$\overline{c.p.}_e = \frac{h}{2} (\mathrm{sgn}\, v_Z)\, \hat{e}_Z + \overline{L} \tag{A-5}$$

$$\overline{c.p.}_c = \frac{\pi R}{4\sqrt{1 - v_Z^2}} \left[v_X \hat{e}_X + v_Y \hat{e}_Y \right] + \overline{L}, \tag{A-6}$$

where R is the radius of the cylindrical portion and h is the height. The component of $\hat{v}$ along the Z axis determines which end is illuminated, hence the coefficient $(\mathrm{sgn}\, v_Z)$ for $\overline{c.p.}_e$. The terms $(\hat{v} \cdot \hat{n})\, \hat{n}$ and $\hat{n} \times \hat{v} \times \hat{n}$ for these surfaces become:

$$(\hat{v} \cdot \hat{n})_e\, \hat{n} = v_Z \hat{e}_Z \tag{A-7}$$

$$(\hat{v} \cdot \hat{n})_c\, \hat{n} = v_X \hat{e}_X + v_Y \hat{e}_Y \tag{A-8}$$

$$(\hat{n} \times \hat{v} \times \hat{n})_e = v_X \hat{e}_X + v_Y \hat{e}_Y \tag{A-9}$$

$$(\hat{n} \times \hat{v} \times \hat{n})_c = v_Z \hat{e}_Z \tag{A-10}$$

Substitution of Eqs. (A-5) through (A-10) into Eq.(2), and combining with (A-3) gives the solar radiation torque acting on the vehicle;

$$\begin{aligned}
\frac{T_x}{V} = {} & L_Z \pi R^2 (1-\nu_e) \left| v_Z \right| v_Y + \frac{h}{2} \pi R^2 (1-\nu_e) v_Z v_Y + L_Z 2Rh(1+\nu_c) v_Y \sqrt{1-v_Z^2} \\
& + 2L_Z A_p (1+\nu_p) v_Y \sqrt{1-v_X^2} - 2L_Y A_p (1+\nu_p) v_Z \sqrt{1-v_X^2} \\
& - L_Y \pi R^2 (1+\nu_e) \left| v_Z \right| v_Z - L_Y 2Rh(1-\nu_c) v_Z \sqrt{1-v_Z^2} \\
& - \frac{\pi R}{4} 2Rh(1-\nu_c) v_Y v_Z \\[2ex]
\frac{T_Y}{V} = {} & -L_Z \pi R^2 (1-\nu_e) \left| v_Z \right| v_X - \frac{h}{2} \pi R^2 (1-\nu_e) v_Z v_X - 2L_Z A_p (1-\nu_p) v_X \sqrt{1-v_X^2} \\
& - L_Z 2Rh(1+\nu_c) v_X \sqrt{1-v_Z^2} + L_X \pi R^2 (1+\nu_e) \left| v_Z \right| v_Z \\
& + 2L_X A_p (1+\nu_p) v_Z \sqrt{1-v_X^2} + \frac{\pi R}{4} 2Rh(1-\nu_c) v_X v_Z \\
& + L_X 2Rh(1-\nu_c) v_Z \sqrt{1-v_Z^2} \\[2ex]
\frac{T_Z}{V} = {} & L_Y 2Rh(1+\nu_c) v_X \sqrt{1-v_Z^2} + L_Y \pi R^2 (1-\nu_e) v_X \left| v_Z \right| + 2L_Y A_p (1-\nu_p) v_X \sqrt{1-v_X^2} \\
& - L_X \pi R^2 (1-\nu_e) v_Y \left| v_Z \right| - 2L_X A_p (1+\nu_p) v_Y \sqrt{1-v_X^2} \\
& - L_X 2Rh(1+\nu_c) v_Y \sqrt{1-v_Z^2}
\end{aligned} \tag{A-11}$$

The expression for the components of $\hat{v}$ along the body axes is obtained by resolving Eq. (3) to the body axes using Eqs. (6) and (7) for ϕ and θ small:

$$\begin{bmatrix} v_X \\ v_Y \\ v_Z \end{bmatrix} = \begin{bmatrix} -\cos\psi \sin(\alpha-\eta) \sin S' + \sin\psi \cos S' - \theta \cos(\alpha-\eta) \sin S' \\ \sin\psi \sin(\alpha-\eta) \sin S' + \cos\psi \cos S' + \phi \cos(\alpha-\eta) \sin S' \\ \theta\left[-\cos\psi \sin(\alpha-\eta) \sin S' + \sin\psi \cos S'\right] + \cos(\alpha-\eta) \sin S' \\ \quad - \phi\left[\sin\psi \sin(\alpha-\eta) \sin S' + \cos\psi \cos S'\right] \end{bmatrix} \tag{A-12}$$

For ideal control as defined by Eqs. (8), (9), and (10), the components of $\hat{v}$ along the body axes become:

$$\begin{bmatrix} v_X \\ v_Y \\ v_Z \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{1 - \cos^2(\alpha-\eta) \sin^2 S'} \\ \cos(\alpha-\eta) \sin S' \end{bmatrix} \tag{A-13}$$

Either Eq. (A-12) or (A-13) may be substituted in Eq. (A-11) to give the expression for solar radiation torques on the vehicle.

APPENDIX 2

Effects of Solar Torques on Spin Stabilized Vehicle

The expression for the solar torques resolved along the sun reference axes is obtained using Eqs. (24), (25), and (A-11) and is given by:

$$\begin{bmatrix} T_{xo'} \\ T_{yo'} \\ T_{zo'} \end{bmatrix} = \begin{bmatrix} \Delta(S') + \lambda_1(S')\left[-L_X \sin(\omega_r t-\eta) - L_Y \cos(\omega_r t-\eta)\right] \\ \lambda_1(S')\left[-L_Y \sin(\omega_r t-\eta) + L_X \cos(\omega_r t-\eta)\right] \\ \lambda_2(S')\left[L_Y \sin(\omega_r t-\eta) - L_X \cos(\omega_r t-\eta)\right] \end{bmatrix} \tag{B-1}$$

where

$$\Delta(S') = -L_Z V \sin S' \left[2Rh(1+\nu_c)\left|\sin S'\right| + \pi R^2(1-\nu_e)\left|\cos S'\right|\right] + \frac{\pi R^2 h V}{2}(\nu_e - \nu_c)\cos S' \sin S' \tag{B-2}$$

$$\lambda_1(S') = \left[\pi R^2(1+\nu_e)\left|\cos S'\right| + 2Rh(1-\nu_c)\left|\sin S'\right|\right] V \cos S' \tag{B-3}$$

$$\lambda_2(S') = \left[2Rh(1+\nu_c)\left|\sin S'\right| + \pi R^2(1-\nu_e)\left|\cos S'\right|\right] V \sin S' \tag{B-4}$$

Substitution of Eq. (B-1) into (12) with $t_o = 0$ and $\overline{H}_{oi} = I_Z \omega_r \hat{e}_{yo'}$ results in:

$$\begin{bmatrix} H_{xo'} \\ H_{yo'} \\ H_{zo'} \end{bmatrix} = \begin{bmatrix} 0 \\ I_Z \omega_r \\ 0 \end{bmatrix} + \begin{bmatrix} \Delta t + \frac{\lambda_1}{\omega_r}\left[L_X \cos(\omega_r t-\eta) - L_Y \sin(\omega_r t-\eta) - L_X \cos\eta - L_Y \sin\eta\right] \\ \frac{\lambda_1}{\omega_r}\left[L_Y \cos(\omega_r t-\eta) + L_X \sin(\omega_r t-\eta) - L_Y \cos\eta + L_X \sin\eta\right] \\ \frac{\lambda_2}{\omega_r}\left[-L_Y \cos(\omega_r t-\eta) - L_X \sin(\omega_r t-\eta) + L_Y \cos\eta - L_X \sin\eta\right] \end{bmatrix} \tag{B-5}$$

The terms with coefficeints $\frac{\lambda_i}{\omega_r}$ are periodic with the spin frequency relative to inertial space, ω_r; hence in n earth orbits, $\Delta t = \Delta \frac{2\pi n}{\omega_o}$. Therefore since $|\lambda_1| \cong |\Delta|$, Eq. (B-5) may be approximated by:

$$\begin{bmatrix} H_{xo'} \\ H_{yo'} \\ H_{zo'} \end{bmatrix} \cong \begin{bmatrix} \Delta t \\ I_Z \omega_r \\ 0 \end{bmatrix} \qquad t \ll 1 \text{ year} \tag{B-6}$$

The average angular momentum change per orbit is then just $\frac{2\pi}{\omega_o} \Delta (S') \hat{e}_{xo'}$. Since the sun reference set is slowly rotating relative to the orbital reference set, the average angular momentum of the vehicle relative to interial space $\overline{H}_i$ becomes:

$$\overline{H}_i = I_Z \omega_r \hat{e}_{yo} + \int_{k=o}^{m \gg 1} \frac{2\pi}{\omega_o} \Delta (S') \left[\cos \eta \, \hat{e}_{xo} - \sin \eta \hat{e}_{zo} \right] dk, \tag{B-7}$$

where k is the number of complete orbits the integration includes, given by:

$$k = t / \frac{2\pi}{\omega_o} \tag{B-8}$$

Substitution of Eqs. (4), (5), (B-2), and (B-8) in Eq. (B-7) gives the average angular momentum $\overline{H}_i$ in inertial space:

$$H_{x\dot{o}}(t) = -\frac{\pi R^2 h V}{4\omega_k} (\nu_e - \nu_c) \sin \xi \, (1 - \cos 2\omega_k t)$$

$$+ \frac{L_Z V (1+\nu_c) 2 R h}{2\omega_k \sin \xi} \left[\sin \xi \, \sin \omega_k t \sqrt{1 - \sin^2 \xi \sin^2 \omega_k t} + \sin^{-1} (\sin \xi \sin \omega_k t) \right]$$

$$+ \frac{L_Z V \pi R^2 (1-\nu_e) \left| \sin \xi \right|}{2\omega_k} \sin \omega_k t \left| \sin \omega_k t \right|$$

$$H_{y_o}(t) = I_Z \omega_r \tag{B-8}$$

$$H_{z_o}(t) = \frac{\pi R^2 h V}{4\omega_k} (\nu_e - \nu_c) \cos\xi \;\; \sin\xi \,(2\omega_k t - \sin 2\omega_k t)$$

$$+ \frac{L_Z V(1+\nu_c) 2Rh}{\omega_k} \cos\xi \left[(\cos\omega_k t - 1)\left(1 - \frac{\sin^2}{2} - \frac{5\sin^4}{6} - \ldots\right) \right]$$

$$+ \frac{L_Z V \pi R^2 (1-\nu_e) \left|\sin\xi\right| \cos\xi}{2\omega_k} \left[-\cos\omega_k t \left|\sin\omega_k t\right| + \left(t - \frac{n\pi}{\omega_k}\right) \frac{\sin\omega_k t}{\left|\sin\omega_k t\right|} \right]$$

where n is even and chosen such that $\left| t - \frac{n\pi}{\omega_k} \right| \leq \frac{\pi}{\omega_k}$.

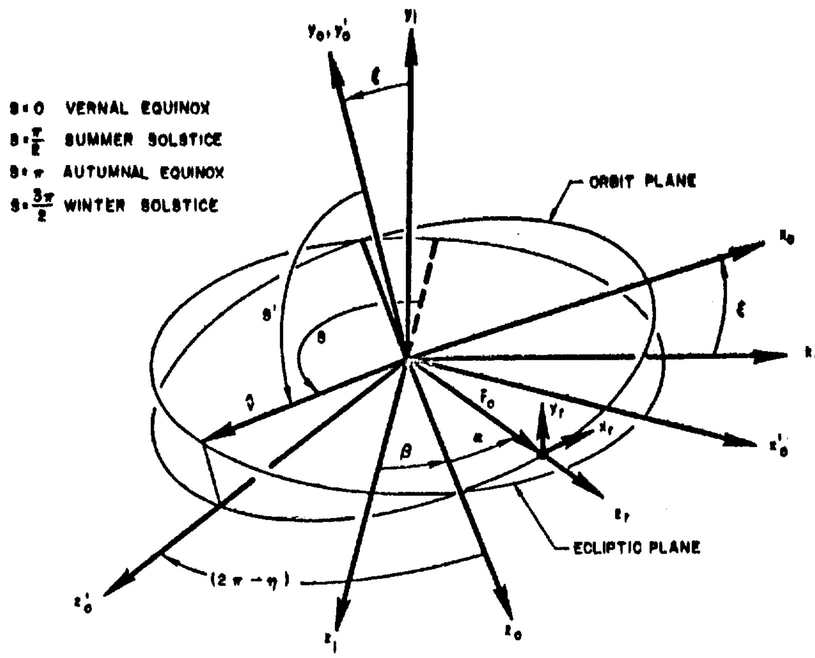

FIGURE 1

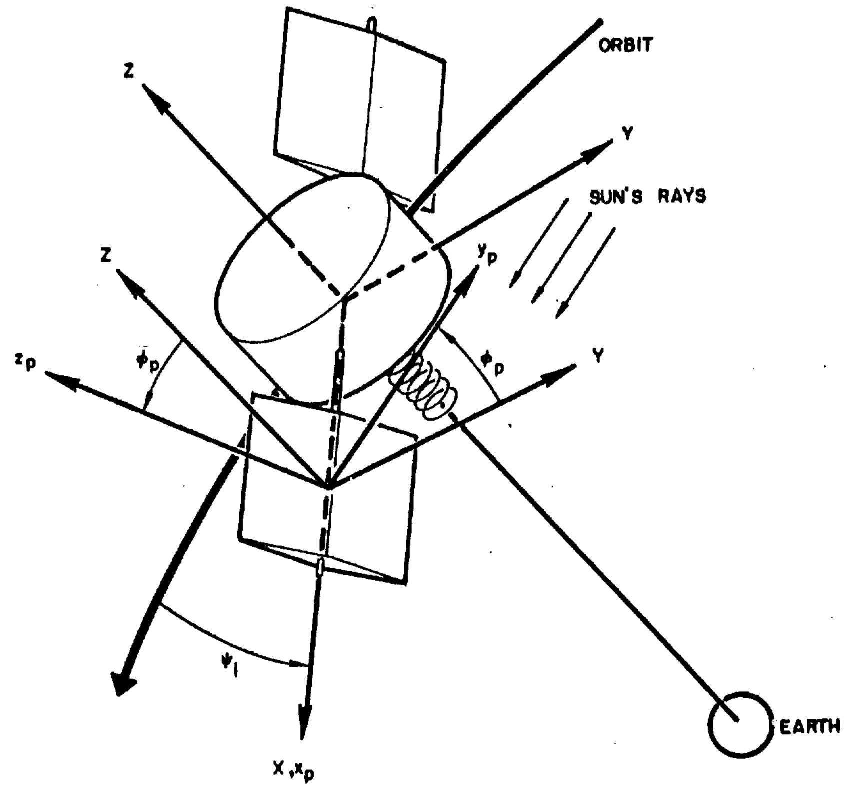

FIGURE 2

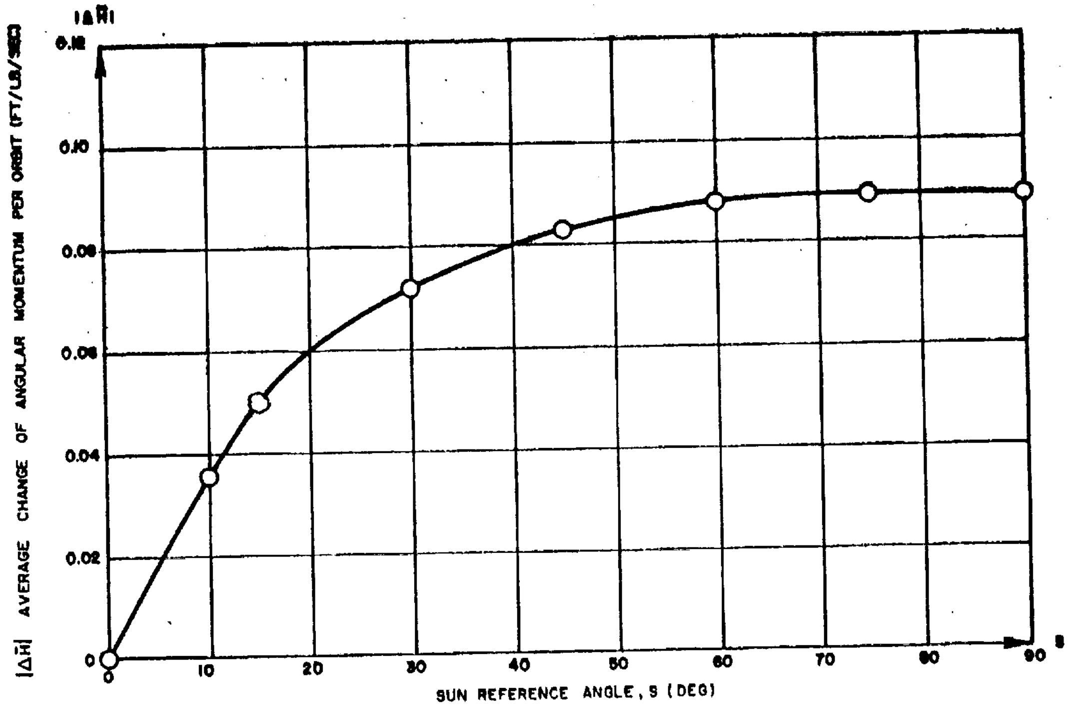

FIGURE 3

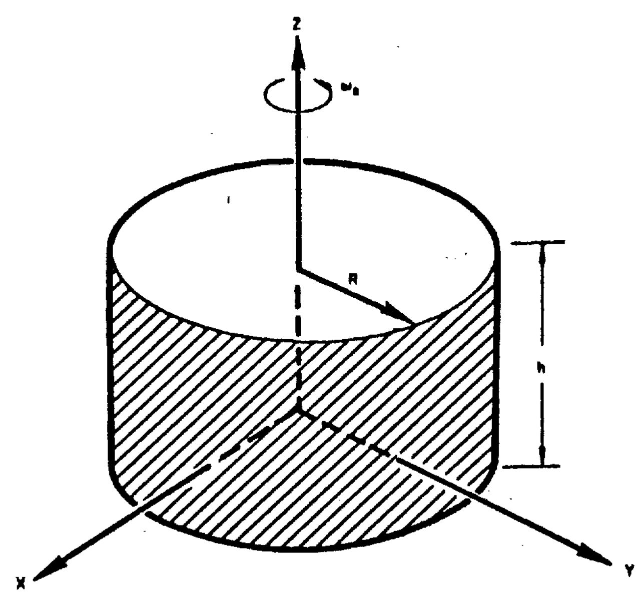

FIGURE 4

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